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Thermal Characterization of a Multilayer Material Through the Flash Method

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The extension of the heat pulse method, commonly used for the measurement of the diffusivity of homogeneous isotropic materials, to the characterization of multiplayer structures that can be found in electronic boards, for example, is examined. Both models and experimental estimations of thermophysical property measurement are presented for the diffusivity in the thickness direction, for interface contact resistance (glue) characterization, and for deposits on substrates. A strong emphasis is placed on sensitivity analysis and the type of thermal parameter that can be reached in practice for a given experiment. Estimation of the in-plane thermal diffusivity of possible anisotropic flat-plate samples through a nonhomogeneous pulse excitation and infrared camera measurements are presented.

Nomenclature

A, B, C, D	=	coefficients of the quadrupole matrix
a	=	thermal diffusivity, $m^2 \cdot s^{-1}$
Bi	=	Biot number
c	=	heat capacity, $J \cdot kg^{-1} \cdot K^{-1}$
e	=	thickness, m
F	=	Fourier cosine transform of f
f	=	space function
g	=	time function
H	=	losses parameter, s^{-1}
h	=	convective heat transfer coefficient, $W \cdot m^{-2} \cdot K^{-1}$
K	=	thermal parameter
L	=	length, m
p	=	dimensional Laplace variable, s^{-1}
Q	=	absorbed fraction of the flux density, $J \cdot m^{-2}$
R_c	=	interface thermal resistance, $m^2 \cdot K \cdot W^{-1}$
T	=	temperature, K
t	=	time, s
X	=	sensitivity coefficient, K
x, y, z	=	space coordinate, m
α, β	=	eigenvalue, m^{-1}
δ	=	Dirac distribution
Θ	=	Laplace–Fourier transform of temperature, $K \cdot s \cdot m$
θ	=	Laplace transform of temperature, $K \cdot s$
λ	=	thermal conductivity, $W \cdot m^{-1} \cdot K^{-1}$
ρ	=	mass density, $kg \cdot m^{-3}$
σ	=	standard deviation
χ	=	parameter

Subscripts

exp	=	experimental
front	=	top face of the sample
i	=	block number
max	=	maximum
rear	=	bottom face of the sample
x, y, z	=	space direction
$\frac{1}{2}$	=	one-half rise of the thermogram

Superscript

$*$	=	reduced

I. Introduction

THE thermal aspects in the design of electronic circuits become increasingly important with the continuous improvement in their integration (decrease in size, increase in temperature maximum value, etc.). In a hybrid circuit, the power electronic components are generally located at the top of a stack of layers whose thermal properties must be optimized to allow the reduction of the overheating. Thus the thermal characterization of such boards[1] and, more generally, of multilayer materials, constitute a prerequisite for a numerical simulation of the temperature field in this type of structure.[2,3]

The heat pulse (flash) method is currently considered a standard for thermal diffusivity measurements. Since the work of Parker et al.,[4] this method has been the subject of many developments linked to parameter estimation methods,[5–7] sensor technical evolutions, acquisition materials, and data processing.[8] The flash method is used here to characterize thermally the constitutive materials of multilayer stacks.

After a short review of the model of the rear face temperature response to a flash excitation, as well as its application to thermal diffusivity estimations based on a sensitivity study, we will show that this method can be adapted for the estimation of various parameters: thermal diffusivity in the plane and in the thickness directions of the sample for very thin materials (opaque or semitransparent[9,10]), thermal contact resistance of particular interfaces (glue, brazed joint, etc.), and thermal diffusivity and conductivity of a thin deposit on a substrate. For each case, a parametric sensitivity study will be developed and experimental results will be shown. All of the materials and the experimental conditions that are presented here correspond to specific substructures of classical power electronic boards (Fig. 1).

A. Thermal Diffusivity in the Thickness Direction

In this section, a brief review of the notion of a sensitivity coefficient of the flash method and of its practical implementation will be presented. Emphasis is also put on the transfer matrix in the Laplace domain. The thermal diffusivity a_z is the diffusivity in the direction normal to the sample faces (thickness direction).

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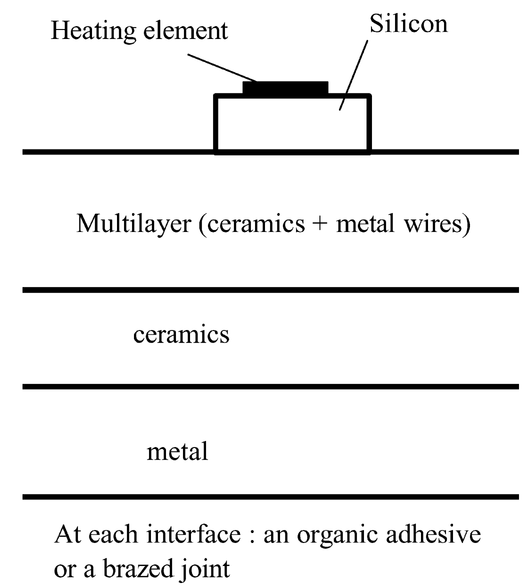

Fig. 1 Typical multilayer structure.

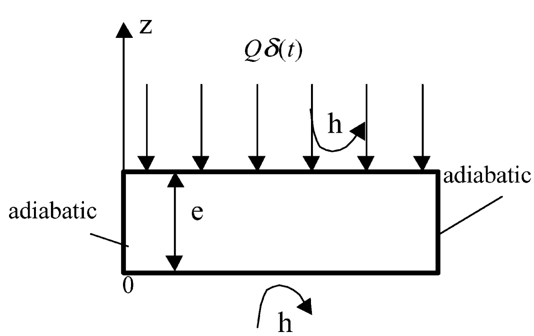

Fig. 2 Flash method experiment.

1. Modeling of the Flash Method

The classical flash experiment (Fig. 2) can be modeled by the thermal quadrupole method.[11] Heat transfer in a material is written in the Laplace transformed time domain by the use of a quadrupole matrix. An h coefficient that represents the losses (natural convection with air, linearized radiation with surrounding solid surfaces, and conductive losses through the sample holder) is used, which gives

$$\begin{bmatrix} \theta_{\text{front}} \\ -h\theta_{\text{front}} + Q \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \theta_{\text{rear}} \\ h\theta_{\text{rear}} \end{bmatrix} \tag{1}$$

where θ is the Laplace transform of temperature T:

$$\theta(z, p) = \int_0^{+\infty} T(z, t) e^{-pt} \, \mathrm{d}t \tag{2}$$

Subscripts front and rear refer to the top or bottom face of the sample, and

$$A = D = \cosh(se), \qquad B = \sinh(se)/\lambda_z s$$
$$C = \lambda_z s \sinh(se) \tag{3}$$

with $s^2 = p/a_z$.

The Laplace transform of the rear face temperature directly stems from Eq. (1):

$$\theta_{\text{rear}} = \frac{\lambda_z s Q}{2h\lambda_z s \cosh(se) + \left(\lambda_z^2 s^2 + h^2\right) \sinh(se)} \tag{4}$$

The return to the time domain can be performed numerically, either through the inverse fast Fourier transform's algorithm or through Stehfest's method (see Ref. 11).

The rear face thermogram $T^*(t)$, normalized with respect to its maximum $T_{\max}[T^*(t) = T(t)/T_{\max}]$, is considered here. Hence, the knowledge of the absorbed fraction of the excitation heat flux density is not required in the parameter estimation problem. The parameters of the model are the characteristic time e^2/a_z and the Biot number Bi ($Bi = he/\lambda_z$).

2. Sensitivity Study and Estimation Method

Once an experimental reduced thermogram $T^*_{\exp}(t)$ is measured, it is possible to estimate the parameters of the model e^2/a_z and Bi by comparison with the theoretical model (4). The measurement of these two parameters is, therefore, indirect, and a parameter estimation problem has to be solved.[12] Sensitivity coefficients can be used to aid in the design of a well-behaved parameter estimation experiment.

The reduced sensitivity coefficients of the reduced temperature $T^*(t)$ to parameters a_z and Bi are

$$X^*_\chi(t) = \chi \frac{\partial T^*(t)}{\partial \chi}, \qquad \text{for} \qquad \chi = a_z \quad \text{or} \quad Bi \tag{5}$$

Noted (Fig. 3) that the sensitivities to the diffusivity and to the Biot number are characterized by maxima that do not occur at the same time. The maximum of the sensitivity to the diffusivity occurs at the time of one-half rise $t_{1/2}$ of the thermogram. (It is exactly this time in the case of absence of heat losses, $Bi = 0$.) Sensitivity to the Biot number is much lower than the sensitivity to diffusivity, even if it increases for long times or with heat losses. If the Biot number is lower than 0.2 but higher than 0.01, the two sensitivities are uncorrelated (nonproportional), and it is, therefore, possible to estimate the thermal diffusivity and the Biot number simultaneously. If the Biot number becomes lower than 0.01, the corresponding sensitivity becomes very small (smaller than 0.01) and the simultaneous estimation of both parameters becomes very difficult for the signal-to-noise ratio met in practice.

The Parker et al. estimation method[4] can be used for cases of very low Biot numbers. This case is seldom met in practice. It corresponds to a low h coefficient or to a thin conductive sample. In practice, the heat losses have to be taken into account. (See, for example, the partial time moments method[13] or the partial time method.[14])

B. Interfaces Characterization

The interface between two layers, a glue or a brazed joint, can be thermally characterized by a contact resistance $R_c = e/\lambda$ if its thickness ($e \leq 100$ μm) is small with respect to the total thickness

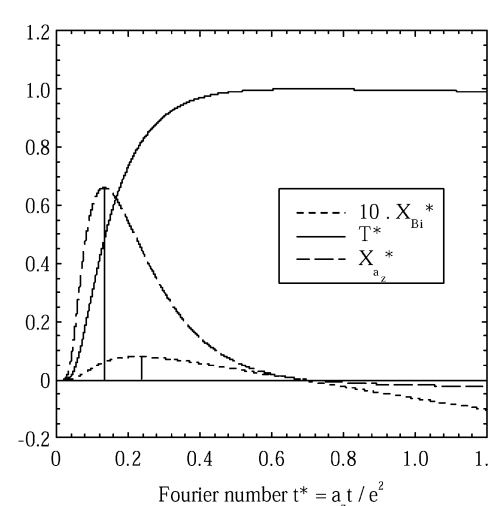

Fig. 3 Normalized rear face thermogram $T^*(t^*)$ and reduced sensitivities of the normalized temperature $X^*_{a_z}(t^*)$ and $X^*_{Bi}(t^*)$: $a_z = 10^{-5}$ $m^2 \cdot s^{-1}$, $e = 5$ mm, and $Bi = 10^{-2}$.

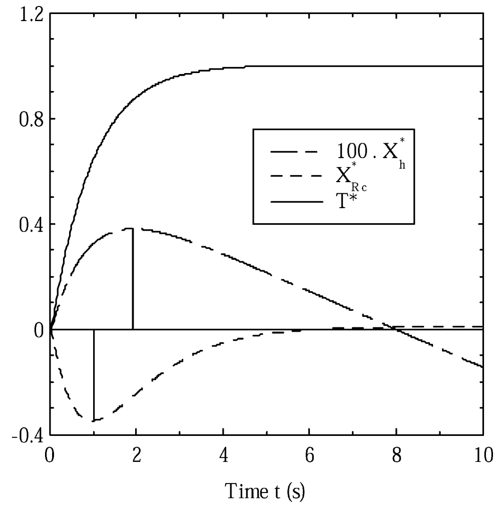

Fig. 4 Flash experiment simulation on a two-layer sample (2-mm brass, 0.18-mm glue, and 2-mm brass) with reduced sensitivities $X^*_{R_c}$ and X^*_h.

of both layers (Fig. 1). The capacitive effect of this interface will, therefore, be neglected.

1. Theoretical Model

If the interface between two layers (1 and 2) is characterized by the use of the concept of a uniform contact resistance R_c, the thermal quadrupole method can be used to model heat transfer through the whole stack.[11] The global expression of heat transfer is simply given by the product of matrices:

$$\begin{bmatrix} \theta_{\text{front}} \\ -h\theta_{\text{front}} + Q \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} 1 & R_c \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} \theta_{\text{rear}} \\ h\theta_{\text{rear}} \end{bmatrix}$$

$$= \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \theta_{\text{rear}} \\ h\theta_{\text{rear}} \end{bmatrix} \tag{6}$$

where subscripts i of A_i, B_i, C_i and D_i correspond to the layer number i, $i = 1$ or 2.

The Laplace transform of the temperature of the rear face can then be easily calculated:

$$\theta_{\text{rear}} = Q/[C + h(A + D) + h^2 B] \tag{7}$$

2. Sensitivity Study

We are interested in the case of a two-layer material. The parameters of the model are the interface resistance R_c and the heat loss coefficient h because the two-layer thermophysical properties and thicknesses are assumed to be known here. The reduced sensitivity coefficients $X^*_\chi(t)$ of the reduced rear face temperature to each parameter $\chi(R_c$ and $h)$ can be calculated by the use of Eq. (5).

From this calculation, one should note that $T_{\max}$, the maximum temperature, also depends on the contact resistance R_c and on the heat transfer coefficient h.

The reduced sensitivity coefficients are plotted in Fig. 4 for a two-layer sample composed of two 2-mm-thick brass slabs stuck together by a 0.18-mm-thick glue ($R_c = 2.6 \times 10^{-4}$ m$^2 \cdot$ K $\cdot$ W^{-1} and $h = 10$ W $\cdot$ m$^{-2} \cdot$ K^{-1}). It is clear that the sensitivity coefficients to R_c and h are uncorrelated: The two sensitivity coefficients are not proportional, and the sensitivity to h is very low. In such a case, it is better to give h a nominal fixed value and to estimate the R_c parameter alone by a least-square method.

C. Deposit Characterization

In some cases, it is not possible to make an experiment on an unknown layer (very thin layer, electrochemically deposited layer, etc.) that is not self-standing. A specific measurement technique different from those actually used for bulk products must then be developed.[15–17]

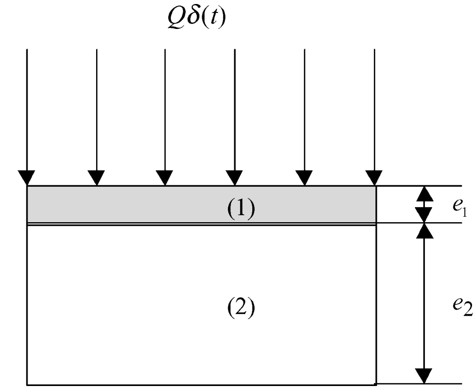

Fig. 5 Thermal characterization of a deposit; principle of the flash method.

1. Description of the Method

Our purpose is to propose a methodology for the estimation of parameters by the use of an extension of the flash method to the nonhomogeneous case of a thin layer, 1, deposited on a thicker self-standing substrate, 2 (Fig. 5). Hence, two experiments have to be made. The first one is made with the sole substrate 2. The second experiment applies to the complete two-layer material, film 1 on substrate 2.

If the properties of the substrate are known, two parameters of the deposited layer have to found. These two parameters must be chosen from among the three following properties: λ_1, ρc_1, and a_1.

The presence of the deposit can be detected with this method if the thermal contrast ΔT^* (difference between the two normalized thermograms) is high enough. If the contrast is weak, only measurements with very low noise or with high absorbed energy will allow the detection of the deposited layer.

The corresponding heat loss model, in the Laplace domain, is given by[9]

$$\Delta\theta^* = \frac{1}{s}\left[\frac{1 + K_1 K_2}{K_2 \sinh(K_1 s)\cosh(s) + \sinh(s)\cosh(K_1 s)} - \frac{1}{\sinh(s)}\right] \tag{8}$$

where $s^2 = pe_2^2/a_2$. The model only depends on two nondimensional parameters chosen from among the four parameters:

$$K_1 = \sqrt{\left[\left(e_1^2/a_1\right)/\left(e_2^2/a_2\right)\right]}$$

the ratio of the square root of the characteristic times;

$$K_2 = \sqrt{\lambda_1 \rho c_1}\Big/\sqrt{\lambda_2 \rho c_2}$$

the ratio of the effusivities;

$$K_3 = K_1 K_2 = \rho c_1 e_1 / \rho c_2 e_2$$

the ratio of the heat capacities; and

$$K_4 = K_1/K_2 = (e_1/\lambda_1)/(e_2/\lambda_2)$$

the ratio of the thermal resistance.

2. Sensitivity Study

The reduced sensitivity coefficients of the thermal contrast $\Delta T^*(t)$ to the two K_i parameters (for example K_1 and K_2) are

$$X^*_{K_i}(t) = K_i \frac{\partial \Delta T^*(t)}{\partial K_i}, \qquad i = 1 \text{ or } 2 \tag{9}$$

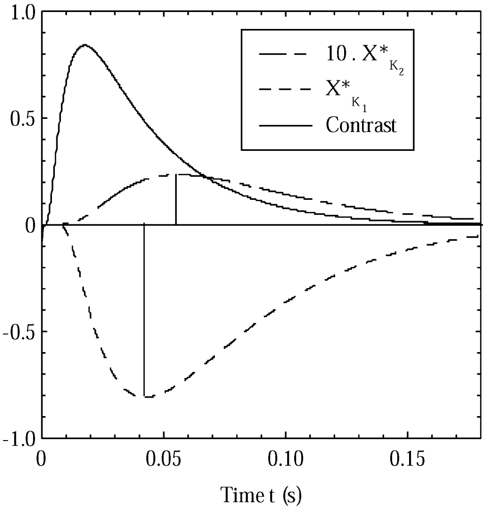

Fig. 6 Heat pulse experiment on a thin layer deposited on a substrate, reduced sensitivities $X^*_{K_1}$ and $10^*X^*_{K_2}$.

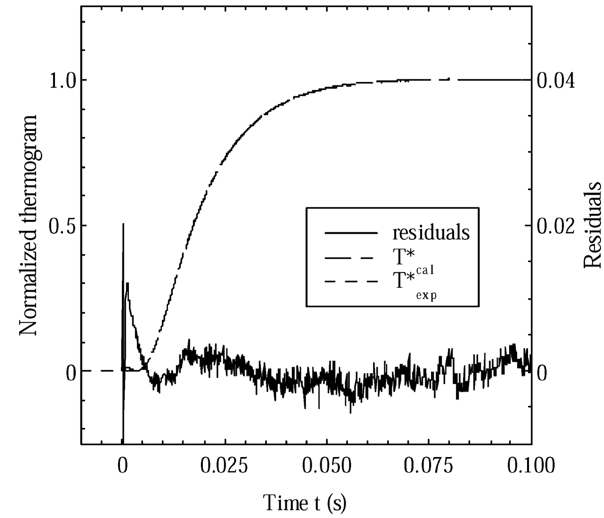

Fig. 7 Flash experiment on a brass sample (*e* = 2 mm).

A precise estimation of the two parameters is possible if the reduced sensitivity coefficients are not proportional. In the case of a two-layer sample characterized by the two K_i ($K_1 = 1.3$ and $K_2 = 0.3$), the variation with time of the sensitivity coefficients (Fig. 6) shows that the maxima of these coefficients occur at slightly different times. If the signal-to-noise ratio is high (higher than 1000), and if the thermal contrast is high (higher than 0.8), both parameters K_1 and K_2 can be identified by a nonlinear least-squares method (the Gauss–Newton method, for example).

The preceding case is quite singular. Generally, the reduced sensitivity coefficients are quasi proportional and only one parameter K_i can be estimated with precision. This parameter is not necessarily K_1 or K_2. It may be K_3, for example, and it is not systematically the same ratio of parameters that can be reached by a flash experiment. Thus, in the case of a resistive film deposited on a metallic substrate, only the deposit's thermal diffusivity given by K_1 can be estimated. In the case of a metallic film deposited on a metallic substrate, only the heat capacity K_3 can be reached.[11]

D. Experimental Characterizations

1. Experimental Setup

Given the very short response times of the materials found in electronic boards, a specific experimental device has been developed. It uses radiative techniques for stimulation (flash lamps) and measurement (infrared sensor). It allows a contactless temperature measurement on the rear face of the sample.

The flash lamps deliver a homogeneous radiation of adjustable energy density (up to 14 $J \cdot cm^{-2}$) in the visible spectral interval through a 40-mm-diam diaphragm. The pulse duration is 100 μs. Once the pulse is triggered, a shutter automatically insulates the flash lamps to prevent any parasitic radiation from the heated tubes, so as not to change the nature of the energy excitation. The flash duration must stay brief enough with respect to the heat diffusion time. In that case, the assumption of a Dirac distribution in time for the heat flux condition imposed on the front face is validated.

These flash lamps are associated with a liquid-nitrogen-cooled infrared detector HgCdTe in the [8–12 μm] spectral interval with a large, sensitive surface (diameter = 2000 μm). This setup is characterized by a very good signal-to-noise ratio. (On average, $T_{max}/\sigma = 2.19/8 \times 10^{-4} = 2737$, where σ is the standard deviation of the output tension of the detector.) The measured temperature is the averaged temperature over the field of view of the detector.

2. Experimental Results

We now present the experimental results related to the characterization of thin and generally highly conductive materials that constitute the different layers of an electrical board.

The samples are painted with a black powder of fairly uniform emissivity to increase both the front face absorbed energy and the temperature signal produced by the infrared signal emission of the rear face. The parameters estimation method uses an electrical tension ratio, which does not require the knowledge of the emissivity of the sample faces, nor the calibration of the detector, provided that it is used in its linearity domain. The thickness of the black powder is about 3 μm. This thickness is small compared to the thickness of the sample (640-μm minimum), and it is assumed to have no influence on the diffusive heat transfer.[18] The initial temperature is the ambient temperature.

a. Isotropic material. Figure 7 shows experimental results obtained on a 2-mm-thick brass specimen. The two parameters, diffusivity and Biot number, are estimated starting from the normalized experimental thermogram according to the partial time moments method.[13] Then, the theoretical thermogram is calculated starting from the estimated values of the parameters: $a_z = 3.15 \times 10^{-5}\ m^2 \cdot s^{-1}$ and $Bi = 1.6 \times 10^{-2}$.

The difference between the experimental and the theoretical thermograms (residuals) is very small and of the same order of magnitude as the noise level (see preceding). One may note, however, that the residuals are weakly correlated for short times ($t < 10^{-2}$ s), which can be caused by some radiation, seen by the detector, and emitted by the surfaces of the sample holder that may have been heated by parasitic radiations of the flash. Apart from this condition, this means that the experiment and the theoretical model fit quite well. The low values of the estimated Biot number show that convective heat transfer can be neglected in that case.

b. Semitransparent material. The sample is made opaque by a thin layer of titanium (0.1 μm), deposited on both faces, and the sample is covered by the black powder later on. Thus, the same estimation method for the thermal diffusivity can be used.[9]

Figure 8 shows experimental results obtained on a 0.640-mm ceramic (aluminium oxide Al_2O_3) specimen. The estimated values of the parameters are $a_z = 7.54 \times 10^{-6}\ m^2 \cdot s^{-1}$ and $Bi < 0$. This negative value for the estimated Biot number simply means that the level of heat losses is very low, which corresponds to a very low sensitivity of the reduced signal to the Biot number. (The noise is responsible for this negative value.) As a consequence, it is legitimate to adopt a zero value for *Bi*. The theoretical thermogram is calculated starting from the estimated values of the parameters. The residuals that are plotted in Fig. 8 exhibit the same behavior as before.

This example shows that proper preparation of the sample faces that dampen radiative effects can allow the estimation of the true (phononic) diffusivity of a semitransparent sample.

c. Estimation of the contact resistance. The interface between two identical layers can be characterized by the use of the method described in Sec. I.B. These two layers are usually joined together by glue or by a brazed joint in electronic boards.

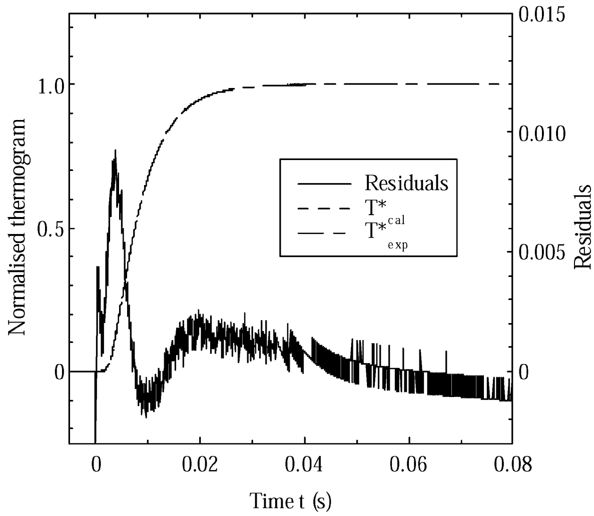

Fig. 8 Flash experiment on a aluminium oxide ceramic Al_2O_3 sample (*e* = 0.64 mm).

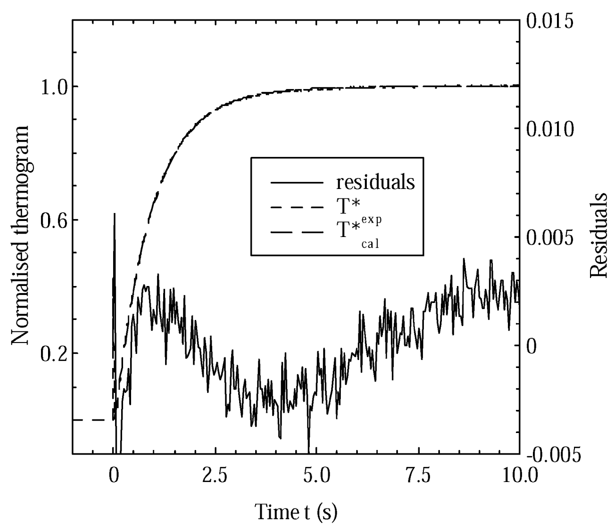

Fig. 9 Experimental and reconstructed thermogram; two-layer brass sample with a glue layer.

The thermophysical properties of the one-layer sample are estimated according to the method described in Sec. I.A for the diffusivity and according to a calorimetry experiment for the estimation of its thermal conductivity ($\lambda = a\rho c$).

Figure 9 shows experimental results obtained for a specimen composed of the following stack: a 1.936-mm brass layer, a 184-μm thick glue, and a 1.944-mm brass layer. With knowledge of the diffusivity of brass (Sec. I.D.1) and its thermal conductivity ($\lambda = 105\ \mathrm{W \cdot m^{-1} \cdot K^{-1}}$), the interface resistance characterizing the glue ($\chi_1 = R_c$), as well as the heat loss coefficient ($\chi_2 = h$) (specific to the experiment), are estimated: $R_c = 2.61 \times 10^{-4}\ \mathrm{m^2 \cdot K \cdot W^{-1}}$ and $Bi < 0$. We then take $Bi = 0$ for the reason just given. The theoretical thermogram is, therefore, calculated starting from the estimated experimental data.

The slightly correlated residuals, whose standard deviation is close to 0.002, are also very low here.

d. Deposit characterization. The sample that has been characterized is composed by the following stack: a 247-μm metallic film deposited on a 640-μm substrate (aluminium oxide ceramic Al_2O_3). Figure 10 shows the rear face response to a flash excitation of the two-layer sample (deposit 1 on substrate 2) and of the substrate. The maximum of the thermal contrast is important (larger than 0.8), and the signal-to-noise ratio is good (normalized standard deviation $\sigma^* = \sigma / T_{\max} = 6 \times 10^{-4}$). Thus, in this case, it is possible to estimate the properties of the film.

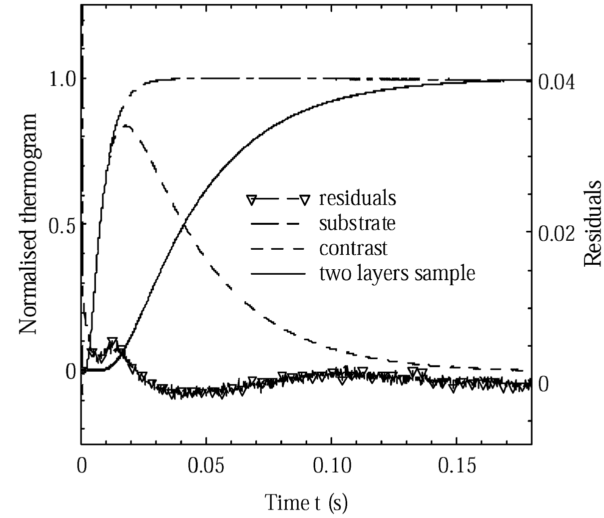

Fig. 10 Thermal characterization of a deposit; experimental heat pulse responses on the substrate and on the two-layer sample: theoretical and reconstructed thermograms.

A flash experiment (results in Sec. I.D.2.b) and a calorimetry experiment allow the determination of the thermophysical properties of the substrate: $c_2 = 0.82 \cdot \mathrm{kJ \cdot kg^{-1} \cdot K^{-1}}$ and $\rho c_2 = 3.08 \times 10^6\ \mathrm{J \cdot m^{-3} \cdot K^{-1}}$. Hence, the thermal conductivity of the substrate is $\lambda_2 = 23\ \mathrm{W \cdot m^{-1} \cdot K^{-1}}$.

Parameters K_1 and K_2 have been estimated, and their values are $K_1 = 1.275$ and $K_2 = 0.321$. The knowledge of the variation with time of the sensitivity coefficients, shown in Fig. 6, allows calculation of the theoretical standard deviation of both parameters ($\sigma_{K_1}/K_1 = 0.02$, $\sigma_{K_2}/K_2 = 0.60$) for the preceding value of the reduced noise σ^*. It confirms that the excellent signal-to-noise ratio allows estimation of both parameters here.

The estimated thermophysical properties of the deposited film are $a_1 = 6.83 \times 10^{-7}\ \mathrm{m^2 \cdot s^{-1}}$, $\lambda_1 = 2.23\ \mathrm{W \cdot m^{-1} \cdot K^{-1}}$, and $\rho c_1 = 3.26 \times 10^6\ \mathrm{J \cdot m^{-3} \cdot K^{-1}}$.

The residuals shown in Fig. 10 for the two-layer experiment have a level that is slightly higher than those in the preceding experiments. This is because they are constructed starting from two experiments, with the corresponding increase in both noise and bias.

E. In-Plane Thermal Diffusivity

The in-plane thermal diffusivity a_x or a_y of a slab is the diffusivity in the directions of the faces of the sample. The classical flash method, with an ideal uniform excitation, allows the measurement of the diffusivity in the direction normal to the faces.

1. Modeling of the In-Plane Flash Method

The principle of the in-plane flash method consists of thermal stimulation of the front face of a parallelepipedic sample of constant thermophysical properties, initially isothermal, by a quasi-Dirac time pulse of heat having any space distribution. In the general case, the heat flux density on the front face can be separated the following way: $\varphi(x, y, t) = g(t) f(x, y)$, where $g(t) = \delta(t)$. The system exchanges heat linearly with the outside environment on its front and rear faces (heat transfer coefficient h), whereas the other faces are assumed to stay adiabatic. The h coefficient is assumed to stay constant in time and uniform in space.

The thermal quadrupole method is used to model the in-plane flash experiment here[19] (Fig. 11). Heat transfer in a one-layer material is written in the Fourier cosine transformed space (x, y) domain and in the Laplace transformed time domain:

$$\begin{bmatrix} \Theta_{\text{front}} \\ -h\Theta_{\text{front}} + F(\alpha, \beta) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \Theta_{\text{rear}} \\ h\Theta_{\text{rear}} \end{bmatrix} \tag{10}$$

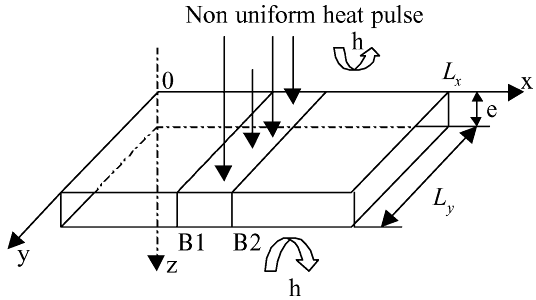

Fig. 11 Principle of the in-plane flash method.

where Θ is the Fourier (cosine)–Laplace transform of temperature T, and F is the Fourier cosine transform of function f:

$$\Theta(\alpha, \beta, z, p) = \int_0^{+\infty} \int_0^{L_y} \int_0^{L_x} T(x, y, z, t) \times \cos(\alpha x) \cos(\beta y) \exp(-pt)\, dx\, dy\, dt \tag{11}$$

$$F(\alpha, \beta) = \int_0^{L_y} \int_0^{L_x} f(x, y) \cos(\alpha x) \cos(\beta y)\, dx\, dy \tag{12}$$

Here, α and β are discrete eigenvalues of the boundary value problem in the (x, y) plane ($\alpha_n = n\pi/L_x$ and $\beta_m = m\pi/L_y$). To have adiabatic lateral faces ($x = 0$, $x = L_x$, $y = 0$, and $y = L_y$), the nonuniform heat flux excitation has to be imposed on the front face, not too close to its sides. The transfer matrix coefficients, as described in Sec. I.A.1, are

$$A = D = \cosh(\gamma e), \qquad B = [\sinh(\gamma e)]/\lambda_z \gamma$$
$$C = \lambda_z \gamma \cdot \sinh(\gamma e) \tag{13}$$

with $\gamma^2 = p/a_z + (\lambda_x/\lambda_z)\alpha^2 + (\lambda_y/\lambda_z)\beta^2$.

The Fourier–Laplace temperature distribution on the rear face can be calculated as

$$\Theta_{\text{rear}} = \frac{\lambda_z \gamma F(\alpha, \beta)}{2h\lambda_z \gamma \cosh(\gamma e) + \left(\lambda_z^2 \gamma^2 + h^2\right) \sinh(\gamma e)} \tag{14}$$

2. Estimation Method and Sensitivity Study

Laplace inversion of Eq. (14) allows the calculation of the Fourier temperature in terms of the average rear face temperature $\theta(0, 0, e, t)$ in the simple Fourier space domain (α, β, e, and t coordinates):

$$\theta(\alpha, \beta, e, t) = \theta(0, 0, e, t) \exp\left[-\left(a_x \alpha^2 + a_y \beta^2\right) t\right] \tag{15}$$

where $\theta(\alpha, \beta, e, t)$ is the Fourier transform of $T(x, y, e, t)$ and a_x and a_y are the thermal diffusivities in the x and y directions.

This yields, for two times t_1 and t_2

$$\frac{\theta(\alpha, \beta, e, t_2)}{\theta(\alpha, \beta, e, t_1)} = \frac{\theta(0, 0, e, t_2)}{\theta(0, 0, e, t_1)} \cdot \exp\left[-a_x \alpha^2 (t_2 - t_1)\right] \cdot \exp\left[-a_y \beta^2 (t_2 - t_1)\right] \tag{16}$$

A simple estimation method consists of a plot of $\ln[\theta(\alpha, 0, e, t_2)/\theta(\alpha, 0, e, t_1)]$ vs $\alpha^2(t_2 - t_1)$. A straight line of slope a_x is obtained. A similar method can be used to obtain a_y. Note that Eq. (16) does not require any assumption for the geometrical form of the excitation [function $f(x, y)$ or $F(\alpha, \beta)$].

For periods much longer than the axial diffusion time $t \gg e^2/a_z$, $\ln[\theta(0, 0, e, t_2)/\theta(0, 0, e, t_1)]$ can be expressed in terms of convective losses only (transient fin hypothesis for which temperature does not depend on the thickness direction z):

$$\ln\left[\frac{\theta(0, 0, e, t_2)}{\theta(0, 0, e, t_1)}\right] = -\frac{2h}{\rho c e}(t_2 - t_1) \tag{17}$$

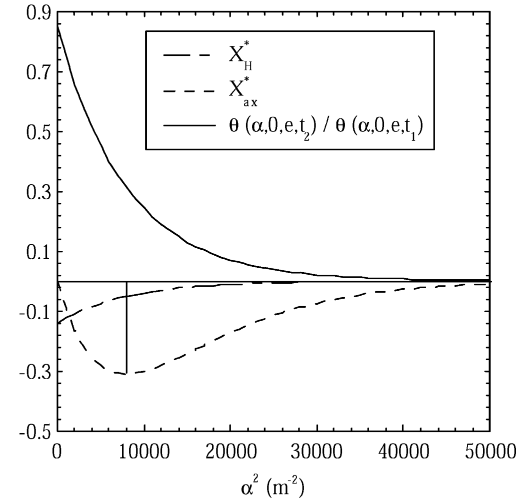

Fig. 12 Sensitivity coefficients X_H^* and $X_{a_x}^*$ of the in-plane flash method: $a_x = 5 \times 10^{-5}$ m$^2 \cdot$ s^1, $H = 66.6 \times 10^{-3}$ s^{-1}, and $t_2 - t_1 = 2.5$ s.

Beside the two diffusivities, the other parameter that can be estimated is

$$H = 2h/\rho c e \tag{18}$$

A sensitivity analysis in a two-dimensional situation corresponding to Fig. 11 with a one-directional excitation corresponding to $f(x, y) = f(x)$, where $f(x)$ is constant on the [B1 B2] interval and to zero elsewhere, shows the influence of parameters H and a_x on the $[\theta(\alpha, 0, e, t_2)/\theta(\alpha, 0, e, t_1)]$ signal (Fig. 12). Parameters H and a_x are uncorrelated, and the maximum sensitivity to H corresponds to $\alpha = 0$, whereas the maximum sensitivity to a_x corresponds to $\alpha^2 = 1/[a_x(t_2 - t_1)]$. This time difference can, thus, be optimized.

The processing of a high number of data points[19] is used and allows a noise reduction of statistical origin.

3. Experimental Validation

a. Experimental apparatus. The experimental bench uses radiative techniques for stimulation, namely, a flash lamp (3-ms pulse duration, 1500 J), and a shortwave infrared camera for measurement. It allows a contactless temperature measurement on the rear face of the sample.

The frames (64 lines of 128 pixels) of the camera are stored at a maximum rate of 25 frames/s. The scanning of one frame is much quicker than the thermal phenomena, and so it is assumed to be instantaneous. The calibration of the distance/pixel number relationship is obtained through observation by the infrared camera of a gold-plated circuit deposited on a glass slab heated by the Joule effect.

The sample is a thin rectangular plate (in-plane dimensions, $L_x \times L_y$; thickness, e; with $L_x \gg e$ to realize $t \gg e^2/a_z$). Hanging the sample on a cantilever through four cotton threads allows a minimization of the lateral losses. The samples are painted with a fairly emissivity black powder whose thickness (a few micrometers) is very small compared to the thickness of the sample. This coating is assumed to have no influence on the diffusive transfer.[18]

b. Experimental results. The experimental results presented here (infrared frame in Fig. 13) correspond to a carbon–epoxy composite plate ($L_x = 40$ mm, $L_y = 40$ mm, and $e = 2$ mm). The recalculated theoretical thermogram with the estimated diffusivity ($a_x = 3.9 \times 10^{-7}$ m$^2 \cdot$ s^{-1}) and the loss coefficient ($H = 4.4 \times 10^{-3}$ s^{-1}) is plotted with an experimental temperature distribution in the x direction at a given time (Fig. 14).

Note that if the estimation of a_x, a_y, and H, given by Eqs. (16) and (17), does not depend on the space distribution of the excitation, it is not the case for the standard deviation of these parameters. This shape can be optimized. (See, for example, Ref. 19.)

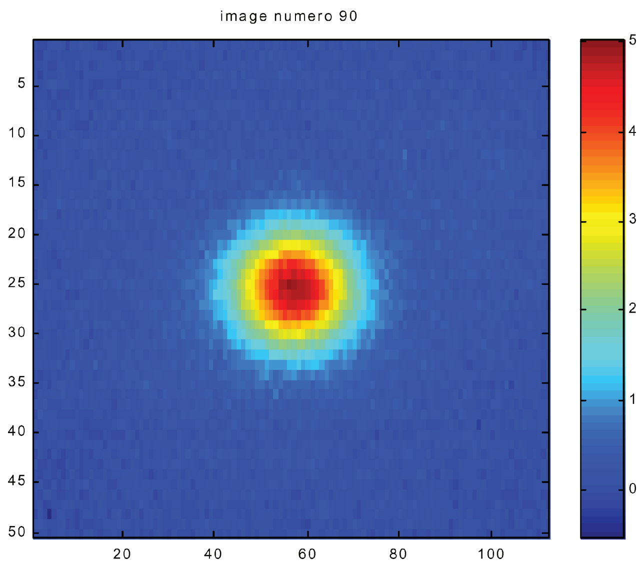

Fig. 13 Experimental temperature field at *t* = 9.32 s, composite material.

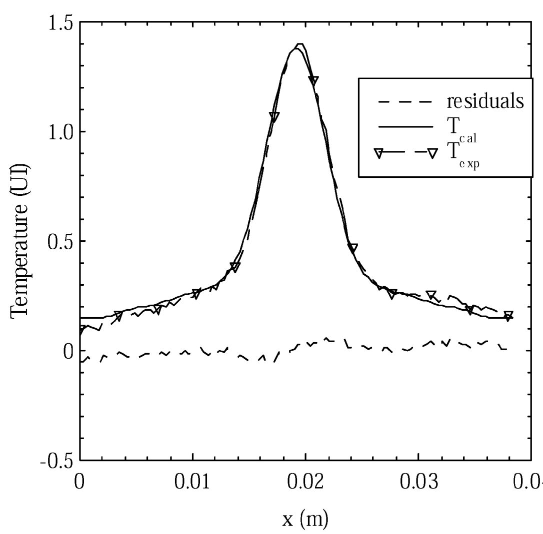

Fig. 14 Experimental and reconstructed estimated *x* distribution of temperature of a composite material at time *t* = 2.4 s.

II. Conclusions

An accurate thermal characterization of the different layers and interfaces of a stack that constitutes a hybrid circuit is made possible by the use of different experimental procedures based on different versions of the flash method. A study of the time variation of the sensitivity coefficients of the rear face thermal response, associated with a plot of the residuals in the corresponding heat pulse experiment, allows for increased confidence in the estimated values of the parameters that are sought.

It has been shown that very good estimation of the thermal diffusivity in the thickness direction of rapid samples, with satisfactory signal-to-noise ratio and residuals (a few thousandths of the maximum signal), could be reached. This technique can be used for some classes of semitransparent samples, provided that a specific material processing of the sample faces is implemented. Estimation by the flash method of a thermal contact resistance in this type of conductive stack is also possible, with experimental residuals of the same quality. The more involved characterization of a deposit on a substrate of known properties is also possible, but special attention must be paid to the type of thermophysical parameters that can be reached by a given pair of experiments.

The flash method can also be extended to the in-plane thermal diffusivity measurement of flat samples (slabs) with an appropriate nonuniform front face excitation and with rear face temperature measurements taken by an infrared camera. The inversion, which uses a model where heat losses are taken into account, is based on the observation of experimental cosine transforms of the signal in the Fourier space domain.

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